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MULTIPLE PERIODIC SOLUTIONS FOR AN ASYMPTOTICALLY LINEAR WAVE E--ETC(U)
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MULTIPLE PERIODIC SOLUTIONS FOR AN ASYMPTOTICALLY LINEAR WAVE EQUATION

K. C. Chang*, S. P. Wu** and Shujie Li***

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ABSTRACT

Two kinds of existence of multiple periodic solutions for an asymptotically linear wave equation are studied. The first concerns the existence of at least three distinct solutions without any group symmetry assumptions on the nonlinear term. The second deals with an estimation of the number of solutions based on the asymptotic behaviour of the nonlinear term at zero and at infinity under an oddness assumption.

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*K. C. Chang, Department of Mathematics, Peking University, Peking, China

**S. P. Wu, Hong Chow University, Hong Chow, China
Courant Institute of Mathematical Science, 251 Mercer Street, New York, New York, 10012

***Shujie Li, Institute of Mathematics, Academia Sinica, Peking, China

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SIGNIFICANCE AND EXPLANATION

Periodic solutions for nonlinear wave equation are studied by many authors in recent years. Most results concern the existence of a non-trivial solution. In this paper, we present two methods to look for more solutions.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the authors of this report.

MULTIPLE PERIODIC SOLUTIONS FOR AN ASYMPTOTICALLY LINEAR WAVE EQUATION

K. C. Chang*, S. P. Wu** and Shujie Li***

In this paper, we study the existence of multiple periodic solutions of the following asymptotically linear wave equation:

$$\begin{aligned} &u_{tt} - u_{xx} + g(t,x,u) = 0 \qquad &(t,x) \in Q = (0,2\pi) \times (0,\pi) \\ &u(0,t) = u(\pi,t) = 0 \qquad &t \in [0,2\pi] \qquad (1) \\ &u(x,0) = u(x,2\pi) & \end{aligned}$$

where $g \in C^1(Q \times R^1, R^1)$, with further additional assumptions.

Two kinds of results are concerned:

(1) The existence of at least three distinct solutions of the equation (0.1). The proof does not depend on any kind of symmetry assumptions on g.

(2) An estimation of the number of solutions of (0.1), based on the asymptotic behaviour of g at $u = 0$ and at $u = \infty$, under an oddness assumption on g.

In case (1), a topological lemma is needed which is due to A. Castro, A. C. Lazer [4] and K. C. Chang [5], and which reads as follows.

Lemma. Suppose that f is a C^2 real valued function defined on R^n. Assume that f is bounded below, satisfies the Palais-Smale condition; and that θ is a nondegenerate, non-minimum critical point of f; then there exist at least three distinct critical points of f.

For simplicity, we express our result in case where g depends on u only. A typical result is the

*K. C. Chang, Department of Mathematics, Peking University, Peking, China

**S. P. Wu, Hong Chow University, Hong Chow, China
Courant Institute of Mathematical Science, 251 Mercer Street, New York, New York, 10012

***Shujie Li, Institute of Mathematics, Academia Sinica, Peking, China,

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Theorem 1. Suppose that $g \in C^1(\mathbb{R}^1)$, satisfies the following conditions:

(g_1) $\beta > 0$ such that

$$0 < g'(u) < \beta, \quad g(0) = 0, \quad g(\pm\infty) = \pm\infty \;;$$

(g_2) $g(u)/u < \gamma < 3$ as $|u|$ large enough ;

(g_3) $g(u)/u > \rho > 3$ as $|u|$ small enough ;

(g_4) $g'(0) \notin \sigma(-\Box) = \{k^2 - j^2 \mid (k,j) \in \mathbb{Z} \times \mathbb{N}^*\}$

where $\mathbb{Z}$ is the integer ring, $\mathbb{N}^*$ denotes the positive integer group; and γ, ρ are constants.

Then the equation (0.1) has at least three distinct solutions.

In case (2), let $f_a = \{x \mid f(x) \le a\}$ and $\gamma(\cdot)$ be the genus of a set. Then according to Clark [6], the two numbers (if they exist),

$$i_1(f) = \lim_{a \to -0} \gamma(f_a)$$

$$i_2(f) = \lim_{a \to -\infty} \gamma(f_a)$$

determine lower bounds for the number of critical points of a functional f, which is even, C^1, satisfies the P.S. condition and $f(\theta) = 0$. Namely, Clark proved that there are at least $(i_1(f) - i_2(f))$ pairs of distinct critical points of f with critical values

$$c_m = \inf_{\gamma(A) \ge m} \sup_{x \in A} f(x) \qquad \text{for } i_2(f) < m \le i_1(f)$$

A typical result in this case is the

Theorem 4. Suppose that $g \in C^1(\mathbb{R}^1)$, odd, satisfies the conditions

(g_1') $g(u)$ is strictly increasing with $g(0) = 0$

(g_2') $\exists\, m$ such that the limit $a = \lim_{u \to \pm\infty} \frac{g(u)}{u} \neq 0$ exists and lies in the interval $(\lambda_m, \lambda_{m+1})$, where $0 < \lambda_1 < \lambda_2 < \lambda_3 < \dots$ are the positive eignevalues of $-\Box$.

(g_3') $\exists\, k$ such that $\lambda_k < g'(0) < \lambda_{k+1}$

Then the equation (0.1) has at least $2|k - m|$ solutions provided $(\lambda_k, \lambda_{k+1}) \cap (\lambda_m, \lambda_{m+1}) = \phi$ and $\lambda_0 = 0$.

The paper is divided into three parts. §1 deals with the case (1), where a combination of the saddle point reduction (eg. cf. [1]), and a "duality" argument due to

Ekeland [7] and Brezis, Coron, Nirenberg [3], is used. §2 presents applications of the Clark criterial to the equation (0.1), our problems again being reduced to finite dimensional ones by the above reduction. §3 consists of remarks, in which we give the generalized formulation of the theorems proved in §1 and §2. Our method can also be used to attack Hamiltonian systems.

We express our grateful thanks to Professor H. Brezis, who suggests the problem in case (1). We also wish to thank Prof. L. Nirenberg and P. Rabinowitz for their encouragement and help.

§1. The proof of Theorem 1.

The proof of Theorem 1 is divided into the following four steps:

1° Let $h = g^{-1}$, then $h \in C^1(\mathbb{R}^1)$, and satisfies:

$$(1') \quad \frac{1}{\beta} < h'(t) < \infty, \quad h(0) = 0 ;$$

$$(2') \quad \frac{h(t)}{t} > \frac{1}{\gamma} > \frac{1}{3} \quad \text{for } |t| \text{ large enough} ;$$

$$(3') \quad \frac{h(t)}{t} < \frac{1}{\rho} < \frac{1}{3} \quad \text{for } |t| \text{ small enough} ;$$

$$(4') \quad h'(0) \notin \left\{\frac{1}{k^2 - j^2} \mid j \neq k, (j - k) \in N^* \times Z\right\} .$$

2° Let $\square$ be the linear differential operator: $\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}$, with domain:
$D(\square) = \{u \in C^2(\bar{Q}) \mid 2\pi \text{ periodic in } t, \ \forall x \in [0,\pi]$ and
$u(0,t) = u(\pi,t) = 0 \ \forall t \in [0,2\pi]\}$, and let A be the self adjoint extension of $\square$ on the Hilbert space $L^2(Q)$. We denote the range of A by $R(A)$, and the null space of A by $N(A)$. Let K be the inverse of A, defined on $R(A) = N(A)^\perp$.

Let

$$H(t) = \int_0^t h(s)ds , \tag{1}$$

then there exists a constant C such that

$$H(t) > \frac{1}{2\gamma} t^2 - C|t| \qquad t \in \mathbb{R}^1 , \tag{2}$$

and

$$H(t) < \frac{1}{2\rho} t^2 \quad \text{as} \quad |t| \quad \text{small enough.} \tag{3}$$

Problem (1) is now reduced to find the critical pints of the functional

$$f(v) = \frac{1}{2} \int_\Omega Kv \cdot v + \int_\Omega H(v) \tag{4}$$

on the Hilbert subspace $R(A)$, or equivalently, to find the solution of the operator equation:

$$Kv + Ph(v) = 0 \tag{5}$$

where P is the orthogonal projection onto the subspace $R(A)$, and $h(\ )$ is the Nemytski operator $v \to h(v(x))$, from $L^2(\Omega)$ into itself.

In fact, if v^* is a critical point of (4), or a solution of (5), letting

$$u^* = (I - P)h(v^*) - Kv^* ,$$

we find

$$u^* = h(v^*) ,$$

and

$$Au^* + v^* = 0 ;$$

i.e.

$$Au^* + g(u^*) = 0 .$$

Since the operator $h(\ )$ is invertible, different v^*'s correspond to different u^*'s.

3° The nonlinear operator $Ph(\ )$ is a potential operator on $R(A)$, with potential $\int_\Omega H(v)$, satisfying:

$$\frac{1}{\beta} \| v_1 - v_2 \|^2 \le (Ph(v_1) - Ph(v_2), v_1 - v_2) .$$

The bounded self-adjoint operator K on $R(A)$ has only finitely many eigenvalues in the interval $\left(-\infty, -\frac{1}{\beta}\right]$ and each is of finite multiplicity.

Suppose that E_λ is the spectrum resolution of K, and let

$$P^+ = \int_{-\frac{1}{\beta}}^{\infty} dE_\lambda , \quad P^- = \int_{-\infty}^{-\frac{1}{\beta}} dE_\lambda$$

where with no loss of generality we may assume that $-\frac{1}{\beta}$ is not in the spectrum of K. The equation (5) now is equivalent to the system:

$$P^+Kv + P^+Ph(v) = 0 \tag{6}$$

$$P^-Kv + P^-Ph(v) = 0 \tag{7}$$

Let $v = (v_1 + v_2)$, where $v_1 = P^+v$, $v_2 = p^-v$. It is not difficult to see that for fixed v_2, $P^+K + P^+Ph(v_2 + \cdot)$ is a strongly monotone operator on the Hilbert subspace $P^+R(A)$. Then there is a continuous function $v_1 = v_1(v_2)$ with $v_1(\theta) = \theta$, which is the unique solution of the equation (6) for fixed $v_2 \in P^-R(A)$. Now the equation (5) is equivalent to the equation

$$Kv_2 + P^-Ph(v_1(v_2) + v_2) = 0 \tag{8}$$

Since the subspace $P^-R(A)$ is of finite dimension, there exists a linear homeomorphism $T : R^N \to P^-R(A)$, $z = (z_1,\ldots,z_n) \to v_2 = Tz = \sum_{i=1}^{N} z_i\phi_i$, where $\{\phi_1,\ldots\phi_N\}$ are the eigenvectors of the operator A which span the whole subspace $P^-R(A)$. Let

$$w(z) = v_1 \circ Tz$$

and

$$v(z) = w(z) + Tz$$

Define a function on R^N:

$$a(z) = \frac{1}{2}\int_\Omega [Kv(z)\cdot v(z) + 2H(v(z))]\,dxdt \tag{9}$$

We shall prove in the next step that

(I) $v(z) \in C(R^N, C(\bar{\Omega}))$

(II) $v(x) \in C^1(R^N, R(A))$ with

$$v'(z) = -[P^+K + P^+Ph'(v(z))]^{-1}P^+Ph'(v(z))T + T$$

(III) $h(v(z)) \in C^1(R^N, R(A))$

(IV) $a \in C^2(R^N)$ with

$$d_z a = T^*[KTz + P^-Ph(v(z))] \ ,$$

$$d_z^2 a = T^*KT + T^*P^-Ph'(v(z))v'(z) \ .$$

Thus, equation (8) is exactly the Euler equation of the function a, i.e. we again reduce the solvability of the equation (5) into a problem of finding critical points of the function a.

On one hand, according to the definition of a and the inequality (2), we find

$$a(z) > \frac{1}{2\gamma} \|v(z)\|^2 - C\|v(z)\| - \frac{1}{6} \|v(z)\|^2 \to +\infty$$

as $\|z\| \to \infty$. The function is bounded below.

On the other hand θ is a critical point of a (see (III)), which is nondegenerate. In fact (cf. (II))

$$v'(\theta) = -(P^+K + P^+Ph'(0))^{-1}P^+Ph'(\theta)T + T$$

so that

$$d_z^2 a(\theta) = T^*(K + P^-Ph'(0))T \qquad (14)'$$

provided by the commutability of the projection operator P^+ and the operator $(P^+K + P^+Ph'(0))^{-1}$. According to the assumption (4) (cf. (4')), $d_z^2 a(\theta)$ is invertible.

Finally, we prove that θ is not a minimum. In fact, choosing z such that $T_z = t\phi_1$, where ϕ_1 is an eigenvector corresponding to the eigenvalue -3 of A, and normalized, so that we have

$$a(z) = \frac{1}{2}\left(h'(0) - \frac{1}{3}\right) t^2 + 0(t^2) < 0$$

for $|t| > 0$ small enough.

Now our conclusion follows from the lemma.

4° Prove

(I) $v(z) \in C(R^N, C(\bar{\Omega}))$

From the equation (6), and the definition of $v(z)$, we have

$$P^+Kv(z) + P^+Ph(v(z)) = 0$$

Let

$$u(z) = (I - P^+P)h(v(z)) - P^+PKv(z)$$

Then

$$u(z) = h(v(z)) \qquad (10)$$

and

$$Au(z) = A(P - P^+P)h(v(z)) - P^+v(z)$$

$$= (AP^-)Ph(v(z)) - P^+v(z) .$$

It is known that $v(z) \in C(\mathbb{R}^N, R(A))$, that h maps $L^2(\Omega)$ into itself continuously, and that $AP^- \in \mathcal{L}(R(A),R(A))$. Combining with the well known property of the wave operator [8]:

$$A^{-1} \in L(R(A), C^{1/2}(\bar{\Omega}))$$

we conclude

$$u(z) = A^{-1}[(AP^-)Ph(v(z)) - P^+v(z)] \in C(\mathbb{R}^N, C(\bar{\Omega})) \ .$$

According to (10), we find $v(z) = g(u(z)) \in C(\mathbb{R}^N, C(\bar{\Omega}))$.

(II) $v(z) \in C^1(\mathbb{R}^N, R(A))$

Notice that the nonlinear operator $v \to h(v)$ has a Gateaux derivative $h'(v)$, which is a multiplication operator on $L^2(\Omega)$. Since $h'(t) \geq \frac{1}{\beta}$, the linear operator $P^+K + P^+Ph'(v)$ is positive definite on the subspace $P^+R(A)$; the inverse $(P^+K + P^+Ph'(v))^{-1}$ is well defined and bounded on $P^+R(A)$ for each fixed $v \in L^2(\Omega)$.

For every $h \in R^N$, let

$$\Delta_h w(z) = w(z + h) - w(z), \quad \Delta_h v(z) = \Delta_h w(z) + Th \ ;$$

We have

$$\|\Delta_h w(z) + [P^+K + P^+Ph'(v(z))]^{-1}P^+Ph'(v(z))Th\| \tag{11}$$

$$\leq C\|[P^+K + P^+Ph'(v(z))]\Delta_h w(z) + P^+Ph'(v(z))Th\|$$

$$\leq C\|\int_0^1 P^+P[h'(v(z) + t\Delta_h w(z) + tTh) - h'(v(z))](\Delta_h w(z) + Th)dt\|$$

$$\leq C_1\|\int_0^1 [h'(v(z) + t\Delta_h v(z)) - h'(v(z))]dt\|_{\mathcal{L}(L^2(\Omega))} (\|h\| + \|\Delta_h w(z)\|)$$

due to the fact that

$$\int_0^1 [P^+K + P^+Ph'(v(z) + t\Delta_h v(z))]\Delta_h v(z)dt$$

$$= P^+Kv(z + h) + P^+Ph(v(z + h)) - P^+Kv(z) - P^+Ph(v(z)) = 0.$$

Since $h \in C^1(\mathbb{R}^1)$ and $v(z) \in C(\mathbb{R}^N, C(\bar{\Omega}))$, for every $\varepsilon > 0$, there exists $\delta > 0$, such that

$$\sup_{0<t<1} \|h'(v(z) + t\Delta_h v(z) - h'(v(z))\|_{L^\infty(Q)} < \varepsilon/C_1 \quad \text{for} \quad \|h\| < \delta \tag{12}$$

Combining (11) with (12), we have

$$\|\Delta_h w(z)\| = O(\|h\|)$$

and thus

$$\|\Delta_h w(z) + (P^+K + P^+Ph'(v(z))]^{-1}P^+Ph'(v(z))Th\| = o(\|h\|)$$

This proves $w \in C^1(\mathbb{R}^N, P^+R(A))$ with

$$w'(z) = -[P^+K + P^+Ph'(v(z))]^{-1}P^+Ph'(v(z))T$$

Therefore $v(z) \in C^1(\mathbb{R}^N, R(A))$ and

$$v'(z) = w'(z) + T$$

(III) $h(v(z)) \in C^1(\mathbb{R}^N, R(A))$

In fact,

$$\begin{aligned}
&\|h(v(z+h)) - h(v(z)) - h'(v(z))v'(z)h\| \\
&\quad \leq \sup_{0<t<1} \|h'(v(z) + t\Delta_h v(z)) - h'(v(z))\|_{\mathcal{L}(L^2(Q))} \|\Delta_h v(z)\| \\
&\quad + \|h'(v(z))\|_{\mathcal{L}(L^2(Q))} \|\Delta_h v(z) - v'(z)h\| .
\end{aligned}$$

Using the inequality (12) again, we get the conclusion, with

$$d_z h(v(z)) = h'(v(z)) \cdot v'(z)$$

(IV) $a \in C^2(\mathbb{R}^N)$ with

$$d_z a(z) = T^*[KT_z + P^-Ph(v(z))] \tag{13}$$

and

$$d_z^2 a(z) = T^*KT + T^*P^-Ph'(v(z))v'(z) . \tag{14}$$

We compute directly,

$$\langle d_z a, q \rangle = (Kv(z) + Ph(v(z)), \langle v'(z), q \rangle) \quad \forall \ q \in \mathbb{R}^N , \tag{15}$$

where $\langle , \rangle$ denotes the duality in $\mathbb{R}^N$, and $(\ , \ )$ denotes the duality in $L^2(Q)$.

However,

$$\langle v'(z),q\rangle = \langle P^{+}w'(z),q\rangle + P^{-}Tq$$

$$Kw(z) + P^{+}Ph(v(z)) = 0$$

Thus

$$Kv(z) \quad Ph(v(z)) = P^{*-}\left(KTz + Ph(v(z))\right) .$$

Substituting into (15), we obtain

$$\langle d_z a,q\rangle = \langle T^*[KT_z + P^{-}Ph(v(z))],q\rangle \quad \forall\ q \in R^N$$

i.e. (13) holds. Differentiating again, we obtain

$$d_2^2 a = T^*KT + T^*P^{-}Ph(v(z))v'(z) .$$

§2. The case $g(u)$ is odd.

Let X be a Banach space. Let $f : X \to R^1$ be a C^1, even function satisfying the Palais-Smale condition; and let $f(\theta) = 0$. We have mentioned the two numbers

$$i_1(f) = \lim_{a\to 0-} \gamma(f_a)$$

$$i_2(f) = \lim_{a\to -\infty} \gamma(f_a)$$

play an important role in estimating the number of critical points of f. Moreover it is easily seen that

(1) If there exists a subspace $Y \subset X$ with codim $Y = j$ and $f|_Y \geq b > -\infty$; then $i_2(f) \leq j$.

(2) If there exists $A \subset \Sigma$, the family of all symmetric subsets with respect to the origin in $X\backslash\{\theta\}$, such that $\gamma(A) \geq m$ and $\sup_{x\in A} f(x) < 0$; then $i_1(f) \geq m$.

In fact, in case (1), let $a < b$, then $f_a \cap Y = \phi$. This implies $Pf_a \subset Y^{\perp}\backslash\theta$, where P is the orthogonal projection onto $Y^{\perp}$. There $\gamma(f_a) \leq \gamma(Pf_a) \leq j$. This proves $i_2(f) \leq j$.

In case (2) there exists $a < 0$ such that $A \subset f_a$. We have $\gamma(f_a) \geq \gamma(A) \geq m$ which implies $i_1(f) \geq m$.

For a C^2 function f defined on R^N, $i_1(f)$ $i_2(f)$ are easily estimated by the asymptotic behaviour of f at θ and at ∞. Let ind (A) denote the maximal dimension of the negative eigenspace, of a symmetric matrix A.

Lemma 1. Suppose that $f \in C^2(R^N, R^1)$, is even, $f(\theta) = 0$, and that $f(x) \to +\infty$ as $\|x\| \to \infty$; then f astisfies the P.S. condition with

$$i_1(f) \geq \text{ind}\,(A_0)$$

where $A_0 = f''(\theta)$, and

$$i_2(f) = 0$$

(Trivial).

Lemma 2. Suppose that $f \in C^2(R^n, R^1)$ is asymptotically quadratic, with an invertible A_∞ such that $\frac{1}{2}(A_\infty x, x)$ is its asymptotics i.e. $\|f'(x) - A_\infty x\| = o(\|x\|)$ as $\|x\| \to \infty$. If further, f is even, with $f(\theta) = 0$. (Without loss of generality, we may assume that $\text{ind}\,(A_0) < \text{ind}\,(A_\infty)$, for otherwise, we consider $-f$ instead of f). Then f satisfies the P.S. condition, with

$$i_1(f) \geq \text{ind}\,(A_0)$$

and

$$i_2(f) \leq \text{ind}\,(A_\infty)$$

The conclusion follows directly from the above two properties: (1) and (2), if the P.S. condition is verified for f.

Suppose that $\{x_n\}_1^\infty \subset R^n$ is a sequence such that $(f(x_n)$ is bounded and that) $f'(x_n) \to \theta$. We shall prove that there exists a convergent subsequence.

Since f is asymptotically quadratic, i.e.

$$\|f'(x) - A_\infty x\| = 0(\|x\|) \quad \text{as} \quad \|x\| \to \infty$$

and A_∞ is invertible, for each $\delta > 0$, there exists a constant R such that

$$\|f'(x)\| \geq \|A_\infty^{-1}\|^{-1}\|x\| - \delta\|x\| \quad \text{as} \quad \|x\| > R$$

Letting $\delta = \frac{1}{2}\|A_\infty^{-1}\|^{-1}$, we have

$$\|f'(x)\| > \delta\|x\| > \delta R \quad \text{as} \quad \|x\| > R$$

Thus, if $f'(x_n) \to \theta$, $\{x_n\}$ must be bounded. Therefore a convergent subsequence exists. Now we return to the equation (0.1). Let

$$3 = \gamma_1 < \gamma_2 < \gamma_3 < \ldots$$

be the positive eigenvalues of the self-adjoint operator $-\Box = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial t^2}\right)$ in $L^2(\Omega)$ (the multiplicity of the eigenvalue is counted) and let $\lambda_0 = 0$.

Theorem 2. Suppose that $g \in C^1(\mathbb{R}^1)$ is odd, and satisfies the conditions $(g_1),(g_2)$ and

(g_3') $\quad g'(0) \in (\lambda_k, \lambda_{k+1})$

Then the equation (0.1) has at least k pairs of solutions.

Proof. All the assumptions in Theorem 1 are fulfilled. According to the proof in §1, the problem is reduced to finite dimensional $a : \mathbb{R}^N \to \mathbb{R}^1$, which is C^2, even and satisfies:

$$a(z) \to +\infty \quad \text{as} \quad \|z\| \to +\infty ,$$

with $a(\theta) = 0$. We look for the critical points of a.

Now the conclusion follows from the Lemma 1, if we know

$$\text{ind } (a''(\theta)) = k .$$

To this end, we see from (14)', that

$$d_2^2 a(\theta) = T^*KT + T^*P^- Ph'(0)T$$

Thus the maximal dimension of the negative eigenspace of $d_2^2 a(\theta)$ is exactly k, i.e. $\text{ind } (d_z^2 a(\theta)) = k$.

Theorem 3. Suppose that $g \in C^1(\mathbb{R}^1)$ is odd, and satisfies the conditions $(g_1),(g_3')$ with some positive integer N such that $\beta \in (\lambda_N, \lambda_{N+1})$ and

(g_5) $\quad$ a constant γ such that $\lambda_N < \gamma < \beta$ and $\inf \{ \int_0^u g(s)ds - \frac{\gamma}{2} u^2 \} > -\infty$.

Then the equation (0.1) has at least $N - K$ pairs of solutions.

Proof. All the assumptions of Theorem 5, Corollary 1 in [5] are fulfilled. The procedure, reducing to the finite dimensional case now is only done by the saddle point reduction. The index of the Hessian of the reduced function at θ is $N - K$ (cf. [2, prop. 7.3]); and the reduced function goes to infinite at ∞.

Theorem 4. Suppose that $g \in C^1(\mathbb{R}^1)$, is odd, and satisfies the conditions $(g_1),(g_3')$ and

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Two kinds of existence of multiple periodic solutions for an asymptotically linear wave equation are studied. The first concerns the existence of at least three distinct solutions without any group symmetry assumptions on the nonlinear term. The second deals with an estimation of the number of solutions based on the asymptotic behaviour of the nonlinear term at zero and at infinity under an oddness assumption.